$T>2.17^{\circ} \mathrm{K}$. It was fitted with an empirical formula,
$\beta^{\prime}\left(2.200^{\circ} \mathrm{K}\right)=0.30 \times 10^{-3}+(72.0+6.66 P)^{-1} \mathrm{~atm}^{-1}$,
to about $1.5 \%$; deviations of the measurements are given in Fig. 3. Also given there is a comparison with $\beta$ derived from density data of Keesom and Keesom ${ }^{1}$ and of Edeskuty and Sherman. ${ }^{5}$ Agreement between the three sets of results seems reasonable and the comparison is valid since no $\lambda$ anomalies exist at this temperature.

A view of Fig. 2 again shows that a compressibility curve between 1.80 and $2.05^{\circ} \mathrm{K}$ parallels the $2.20^{\circ} \mathrm{K}$ curve at low pressures, but with increasing pressure it rises above the $2.20^{\circ} \mathrm{K}$ curve, reaching a peak at $P_{\lambda}$. At $P>P_{\lambda}$, the values of $\beta$ drop continuously and approach the $2.20^{\circ} \mathrm{K}$ value. The minima in the $\beta$-versus- $P$ curves increase in depth and breadth as $P_{\lambda}$ increases, but they seem flattest in the middle of the $P_{\lambda}$ range. Although the peaks become more distinct with increased $P_{\lambda}$, the sharpness of all the peak tips required a


Fig. 4. Compressibility of liquid $\mathrm{He}^{4}$ at 1.95 and $2.200^{\circ} \mathrm{K}$.
higher than normal resolution; therefore the pressure increment of a measurement was reduced from the usual 0.27 to 0.05 atm in the vicinity of the peak. Portions of the 1.95 and $1.80^{\circ} \mathrm{K}$ curves are shown in Figs. 4 and 5, respectively, along with the $2.20^{\circ} \mathrm{K}$ values for comparison. The peak at $P_{\lambda}$ fades away with increasing temperature until it almost disappears at $2.05^{\circ} \mathrm{K}$, although the compressibility excess over the $2.20^{\circ} \mathrm{K}$ value is still obvious.

In the region between 1.60 and $1.75^{\circ} \mathrm{K}$, no $\lambda$ transition occurs. However, the results in Fig. 6 show that a minimum in the $\beta$-versus- $P$ curve persists down to $1.70^{\circ} \mathrm{K}$; at $1.75^{\circ} \mathrm{K}$, the rate of rise beyond the minimum is similar to that at $1.80^{\circ} \mathrm{K}$. Below $1.70^{\circ} \mathrm{K}$, (the $1.65^{\circ} \mathrm{K}$ curve is omitted for clarity) the minimum disappears, but a compressibility excess over the $2.20^{\circ} \mathrm{K}$ curve remains, amounting to $15 \%$ at $1.60^{\circ} \mathrm{K}$ near the melting pressure.

The temperature variation of $\beta$ at constant pressure changed according to the proximity of $(T, P)$ to $\left(T_{\lambda}, P_{\lambda}\right)$. For all temperatures, $\beta$ at $P \ll P_{\lambda}$ increased with in-


Fig. 5. Compressibility of liquid $\mathrm{He}^{4}$ at 1.80 and $2.200^{\circ} \mathrm{K}$.
creasing temperature. Near the $\lambda$ transition, the variation of $\beta$ with temperature became inverted. The reversion of $(\partial \beta / \partial T)_{P}$ to the normal plus sign at $P \gg P_{\lambda}$ was not indicated-the compressibilities for different temperatures merged to a common value within $\sim 2 \%$, the experimental error, at the highest pressures.
The accuracy of the measurements is summarized here. From a straight sum of possible individual errors in cell calibration plus those from readings of $\Delta V$ and $\Delta P$, the maximum error in an individual $\beta$ should be 2.5 to $5.0 \%$ for high to low values of $\beta$, respectively; from the root mean square of individual errors, a probable error in $\beta$ is 1.5 to $3.0 \%$ for high to low values. Consideration of $\Delta P_{L} / \Delta P_{U}$ alone in Eq. (2) leads to a precision error of 1.0 to $1.7 \%$ for high to low values of $\beta$. Near the $\lambda$ transition, the decrease in $\Delta P$ for greater resolution probably lowered the accuracy, but here we are mainly interested in the reproducibility of results over a short range of pressure and time. Error in these results is estimated at $2 \%$.

## IV. DISCUSSION

The present compressibility measurements provide a view of normal and abnormal behavior in liquid $\mathrm{He}^{4}$


Fig. 6. Compressibility of liquid $\mathrm{He}^{4}$ at several temperatures below 1.76 and at $2.200^{\circ} \mathrm{K}$.

Table I. Compressibility minima in liquid $\mathrm{He}^{4}$.

| $T$ <br> $\left({ }^{\circ} \mathrm{K}\right)$ | $\beta_{\text {min }}$ <br> $\left(10^{-3} \mathrm{~atm}^{-1}\right)$ | $P\left(\beta_{\text {min }}\right)$ <br> $(\mathrm{atm})$ |
| :---: | :---: | :---: |
| 2.050 | 8.20 | $10.3 \pm 0.3$ |
| 2.000 | 7.42 | $13.7 \pm 0.5$ |
| 1.949 | 6.75 | $17.0 \pm 0.7$ |
| 1.899 | 6.35 | $19.0 \pm 1.0$ |
| 1.880 | 6.27 | $20.0 \pm 1.0$ |
| 1.865 | 6.15 | $20.5 \pm 0.5$ |
| 1.799 | 5.65 | $23.0 \pm 0.5$ |
| 1.750 | 5.35 | $24.5 \pm 0.5$ |
| 1.339 | 5.27 | $250 \pm 0.5$ |
| 1.700 | 5.07 | $26.0 \pm 0.5$ |

through pressure variations. Generally, $(\partial \beta / \partial P)_{T}$ is negative because of the increase in intermolecular repulsive force. In this sense, the present results show liquid $\mathrm{He}^{4}$ is normal for all pressures at $T>2.17^{\circ} \mathrm{K}$. In particular, the liquid at $2.200^{\circ} \mathrm{K}$ seems to have a high degree of normalcy, as here $\beta$ versus $P$ closely follows Tait's relation

$$
\begin{equation*}
V \beta=J(L+P)^{-1}, \tag{4}
\end{equation*}
$$

where $V$ is the corrected molar volume of Edeskuty and Sherman ${ }^{5}$ and $J=3.390$ and $L=8.47$ are empirical constants. This relation fits a wide variety of liquids and was given a fundamental basis for liquids in general by Ginell. ${ }^{7}$
At $T<2.17^{\circ} \mathrm{K}$, the sign of $(\partial \beta / \partial P)_{T}$ changes as $P \rightarrow P_{\lambda}$ from below. The minimum shown in $\beta$ versus $P$ is lacking in the curves of specific heat and thermal expansion versus temperature, which simply continue the trends set by the low-temperature portions of their curves, albeit at accelerated rates. The minima in the $\beta$-versus- $P$ curves follow a regular pattern for both


Fig. 7. Phase diagram of $\mathrm{He}^{4}$ showing the melting curve, the $\lambda$ line, the locus of zero expansion coefficient, and the locus of minimum in compressibility.
${ }^{7}$ R. Ginell, J. Chem. Phys. 34, 1249 (1961).
$\beta_{\text {min }}$ and $P\left(\beta_{\text {min }}\right)$. The values given in Table I show that $\beta_{\min }$ decreases linearly with increasing $P\left(\beta_{\min }\right)$. In the phase diagram of Fig. 7 are shown the locus of $\beta_{\text {min }}$ and the locus of zero thermal expansion, determined by Grilly and Mills. ${ }^{3}$ These two loci indicate a sizable area of anomalous behavior in the $P-V-T$ relations. Goldstein ${ }^{8}$ gave a possible explanation for the value of $(\partial \beta / \partial P)_{r}>0$ as $P$ increases toward $P_{\lambda}$ : The exchange-energy density, decreasing rapidly as the number of normal atoms increases with pressure, provides a net decrease in energy density, which is measured by $1 / \beta$. The same mechanism could account for the minima shown at $T<1.76^{\circ} \mathrm{K}$, where a $\lambda$ transition is cut short by the formation of solid.

Near $P_{\lambda}$, the variation of $\beta$ with $P$ is best expressed by a logarithmic fit

$$
\begin{align*}
10^{3}\left(\beta_{T}-\beta_{2.2}\right) & =a_{-}-b-\log _{10}\left|P-P_{\lambda}\right| \text { for } P<P_{\lambda} \\
& =a_{+}-b_{+} \log _{10}\left|P-P_{\lambda}\right| \text { for } P>P_{\lambda} . \tag{5}
\end{align*}
$$

Here, $\beta_{T}$ and $\beta_{2.2}$ are the measured compressibilities


Fig. 8. $\beta_{T}-\beta_{2.2}$ versus $\log \left|P-P_{\lambda}\right|$ for liquid $\mathrm{He}^{4}$ at several temperatures. The upper curve is for $P<P_{\lambda}$ and the lower curve is for $P>P_{\lambda}$ at each $T$.
at $(T, P)$ and at $\left(2.200^{\circ} \mathrm{K}, P\right)$, respectively, and $P$ is in atmospheres. The constants $a, b$, and $P_{\lambda}$ were determined from plots of $\beta_{T}-\beta_{2.2}$ versus $\log \left|P-P_{\lambda}\right|$. Some graphical examples are given in Fig. 8, while the constants are given in Table II. We see that the linear plots become more definite as the temperature is decreased, or as the $\lambda$ transition of $\beta$ is accented. At the lowest observed $T_{\lambda}$ values, 1.86 and $1.80^{\circ} \mathrm{K}$, Eq. (5) appears to hold for $5 \times 10^{-2}<\left|P-P_{\lambda}\right|<10 \mathrm{~atm}$. This resembles the linear functions of $\log \left|T-T_{\lambda}\right|$ fitted to the thermal expansion, $\alpha_{P}=(1 / V)(\partial V / \partial T)_{P},{ }^{9-12}$ and

[^0]
[^0]:    ${ }^{8}$ L. Goldstein, Phys. Rev. 140, A1547 (1965).
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